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SOLVING BOUNDARY VALUE PROBLEMS OF THERMAL CONDUCTIVITY IN DEGENERATING FUNNEL-SHAPED REGION

ABSTRACT
of the dissertation for the degree of Doctor of Philosophy (PhD) in the educational program 8D05401-Mathematics

Relevance of the topic. Recently, due to the intensive development of modern contact technology and the high speed of electrical devices, the task of reliably measuring the temperature field of the contact system has become increasingly important. The accuracy of such measurements is necessary to ensure stable operation of devices and prevent overheating of components.

At the same time, the constant expansion of the scope of application of contact technology poses a challenge for researchers to optimally select the parameters of contact materials and their operating modes. That is why the study of thermophysical processes occurring in electrical contacts is of particular importance in such areas as automation, instrument engineering, welding technology, electrical equipment, and other devices where contact elements play a key role.

However, for a complete understanding of these processes, it is important to consider not only the static temperature distribution, but also the dynamics of its change over time. When studying the temperature field of high-precision contacts, it is necessary to take into account the change in the size of the contact area caused by the action of electrodynamic forces and the melting of the material at high temperatures.

The moment when the electrodes separate is particularly significant: the contact surface heats up to the melting point, and a liquid metal bridge forms between the electrodes. As the electrodes continue to separate, this bridge breaks apart, causing some of the contact material to transfer from one electrode to the other. This phenomenon, known as bridge erosion, can disrupt the normal operation of electrical devices and reduce their reliability.

By modeling the thermophysical properties of such a bridge, S.N. Kharin arrived at a boundary value problem in which there is no solution region at the initial moment of contact opening. From a mathematical point of view, the peculiarity of the problem under consideration lies precisely in the presence of a moving boundary and the degeneration of the solution region at the initial moment of time. These peculiarities, considered in some of S.N. Kharin's research results, influenced the integral equation of the boundary value problem, and it turned out that the sequence of Picard approximations of the integral equation diverges.

From both a practical and theoretical point of view, boundary value problems of heat conduction in regions that degenerate at the initial moment of time are of particular interest, since classical methods of mathematical physics are not applicable to problems of this type. This is due to the fact that it is impossible to

reconcile the solution of the heat conduction equation with the movement of the boundary of the heat transfer region. To solve thermal problems of this type, it is necessary to use generalized thermal potentials and reduce the initial boundary value problem to a singular integral equation of the Volterra type.

Therefore, the study of boundary value problems in regions that degenerate at the initial moment of time is a topical scientific problem, and this dissertation investigates new boundary value problems for heat conduction equations in degenerate regions with moving boundary conditions. From a mathematical point of view, the specificity of the problems under consideration is as follows: first, the boundary of the domain in which the solution is sought is moving; second, at the initial moment of time, the contacts are in a closed state, and the domain of the solution degenerates to a point. The peculiarities of the problems under consideration lead to the need to study the solvability of singular integral equations of the Volterra type second kind.

Purpose of research. The main purpose of the study is to investigate the solvability of boundary value problems for heat conduction equations with moving boundary conditions in regions that degenerate at the initial moment of time, as well as to study solutions of singular integral equations of the Volterra type second kind.

Research objectives:

- formulation of a new boundary value problem with moving boundary conditions for solving a two-dimensional parabolic problem in a domain that degenerates at the initial moment of time;
- describing the given functions and solution spaces of the problems under consideration;
- transforming the initial problems;
- reducing the boundary value problems to a singular integral equation of the Volterra type;
- constructing and solving characteristic integral equations;
- evaluation and investigation of resolvent and kernel;
- solution of initial integral equations using the equivalent regularization method;
- solution of initial boundary value problems.

Research methods. The work uses methods from the general theory of partial differential equations, the general theory of differential equations, and functional analysis, as well as methods of Laplace and Fourier integral transforms, special functions, and functions of a complex variable.

Scientific novelty. The paper proposes new boundary value problems with moving boundary conditions for solving two-dimensional parabolic problems in domains that degenerate at the initial moment of time, and investigates ways to solve them.

Theoretical and practical significance of the research. The results obtained in this work are theoretical in nature and make a significant contribution to the development of the theory of boundary value problems for parabolic equations in domains with time-varying boundaries, as well as occupying an important place in

the theory of Volterra-type integral equations. The results of the study can be used in teaching special courses in mathematics at higher educational institutions.

Main results presented for defense:

1. solving new boundary value problems with boundary conditions for heat conduction equations in regions with moving boundaries that degenerate at the initial moment of time;
2. transforming problems into singular integral equations of the Volterra type;
3. construction of characteristic integral equations;
4. construction of resolvent and kernel estimation;
5. solving the initial integral equations by the equivalent regularization method;
6. theorems on the solvability of given boundary value problems.

Reliability and validity of the results. The constructive nature of the methods used in the work ensures the reliability and validity of the research. General provisions are formulated in the form of theorems, for which corresponding proofs are provided.

Publications. The main results of the dissertation are published in 10 scientific papers, including 3 articles in journals included in the Web of Science Core Collection and Scopus databases, and 7 abstracts in the proceedings of international scientific conferences.

Approval of the work. The main results of the dissertation research were presented and discussed at the following conferences and seminars:

- International Scientific and Practical Conference "Trends in the Development of Modern Mathematics and Its Teaching in the Context of Digitalization of Education" (April 27-28, 2023, Academician A. Kuatbekov University of Friendship of Peoples, Shymkent, Republic of Kazakhstan);
- International Scientific and Practical Conference dedicated to the 100th anniversary of the birth of Corresponding Member of the Academy of Sciences of Kazakhstan, Doctor of Physical and Mathematical Sciences, Professor T.I. Amanov "Analysis, Differential Equations and Their Applications" (June 22-23, 2023, Kazakhstan branch of Lomonosov Moscow State University, Astana, Republic of Kazakhstan);
- VII World Congress of Turkic Mathematicians (September 20-23, 2023, Turkestan, Republic of Kazakhstan);
- VII International Scientific Conference "Nonlocal Boundary Value Problems and Related Problems in Mathematical Biology, Computer Science, and Physics" (December 4-8, 2023, Institute of Applied Mathematics and Automation, KBNTS RAS, Nalchik, Russian Federation);
- Traditional International April Conference (April 16-19, 2024, Institute of Mathematics and Mathematical Modeling, SC MSHE, Almaty, Republic of Kazakhstan);
- XI International Conference "Modern Problems of Mathematics and Mechanics," dedicated to the memory of Azerbaijani scientist and thinker Nasir al-Din al-Tusi (July 3-6, 2024, Baku, Azerbaijan);

- International Scientific Conference “Nonclassical Equations of Mathematical Physics and Their Applications” (October 24-26, 2024, Mirzo Ulugbek National University of Uzbekistan, Tashkent, Republic of Uzbekistan);
- Seminar of the Institute of Applied Mathematics of the Karaganda National Research University named after Academician E.A. Buketov.

The doctoral candidate’s contribution to the preparation of each publication. In 10 works completed jointly with academic supervisors and co-authors, the academic supervisors set the task, while the doctoral student independently formulated the main and additional results and presented evidence for them.

Structure and volume of the dissertation. The dissertation, which is 91 pages long, consists of the following structural elements: introduction, two sections, conclusion, and list of sources used. Mathematical statements (theorems, lemmas, remarks) and formulas are numbered with three digits: the first digit indicates the chapter number, the second indicates the section number, and the third indicates the serial number of the corresponding mathematical statement or formula.

Number of sources used - 83.

Keywords: Boundary value problem for heat conduction, moving boundary, domain, singular Volterra integral equation, integral transforms, Carleman - Vekua regularization method.